quantil Matemáticas Aplicadas

**A Reply to Industry Comments:**

# An Evaluation of CREG 051 – 2009 Regulatory Intervention in Colombian Electricity Market

By

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## Introduction

Given time constraints, the following document tries to answer as many comments as possible to our first version (for comments) of our final report: An Evaluation of CREG 051 – 2009 Regulatory Intervention in Colombian Electricity Market, November 19, 2013. In doing so, we refer constantly to the new version of the report (version 2, dated December 20, 2013) that extends considerably the first version of the report and tries to answer as technically as possible some of the most relevant comments.

Before replying to the individual questions and comments, we would like to begin with a general statement that would guide the rest of the discussion. When anyone tries to study reality and reach conclusions about it, one needs to make assumptions that are necessarily restrictive and not completely in accord with reality. In choosing such assumptions, one tries to approximate reality as good as possible and make sure that the approximations would not affect the final conclusions. Everything that we did in this project follows this general principle. Of course, we cannot be completely sure that all results are robust to our assumptions. But we did test the robustness of our models and, at this point, we strongly believe that all of our conclusions would not change even if we did not make the approximations that we did.

On the other hand, the majority of received comments just point out some of these approximations and conclude that our results are not sound. This is a fallacy of argumentation: the fact that an assumption is not completely true does not imply that the conclusion is false. To contradict our conclusions, one would need to write down a model and show that the introduction of a better approximation would lead to different results.

In spite of the above, as an answer to the comments, we conducted more tests and checked whether the pointed facts would change the conclusions. Invariably, we obtained the same qualitative result as the original. This reinforced our confidence in the results, as stated above.

## Answers to ACOLGEN

### A. General comments.

- Market intervention between years 2009 and 2010. Please see section 6(d) on page 97 the final report (Version 2 December 20).

- The role of contracts on consumer welfare, please see section 6(e) page 99.

- The study by FEDESARROLLO is dated 2009 we don't see the point.

- The study by Barrera and Garcia is dated 2010. To understand better the nature of the claim it is important to specify what the X is in their model. An ARMAX model for the conditional mean is a univariate ARMA models with exogenous variables. We need to know what the specific exogenous variables they used but obviously there are many variables that are correlated with the spot price and we have experience doing such models. We assume that the comment is meant to say that we should consider such variables in the analysis. A proxy for that is el Niño – Niña although *aporte rios* might be much better. We do not believe that this will overturn our results, but we will explore it in a new version of the econometric study that will be available shortly.

- The study by ECSIM. This same study has identified several periods of considerable exercise of market power.

  *…,hay al menos tres períodos sostenidos en que la diferencia entre los precios reales y los precios de referencia competitivos es sustancial. Estos períodos también coinciden con los períodos en que los tres mayores proveedores del mercado colombiano tienen una capacidad significativa de ejercer poder unilateral de mercado.*

  *La existencia de estos períodos sostenidos de precios de mercado reales significativamente más altos que los precios de referencia competitivos es consistente con el patrón típico de ejercicio de poder unilateral de mercado en las industrias de oferta de electricidad dominadas por hidroeléctricas.*[1]

  Also, see presentacion de propuestas el sector electrico. ECSIM, August 2013, section 5.1 :

  *Hay por lo menos tres períodos sostenidos de dos a seis meses de duración en los cuales la diferencia promedio entre los precios reales y los precios de referencia competitivos es muy grande. Debido a que estos períodos persisten por más de tres meses, las rentas totales de poder de mercado en cada uno de ellos son significativas económicamente.*

  Other studies or presentation sthat have arrived to similar conclusions: Wolak, F. (2009). "Report on Market Performance and Market Monitoring in the Colombian Electricity Supply Industry", Estudio para la SSPD. Julio.

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[1] Measuring Performance in Colombian Wholesale Electricity Supply Industry. Frank A. Wolak. 2013.

Wolak, F (2010). “Market Performance in the Colombian Electricity Market and the Recent El Nino Event”. PP presentation. October.

**Marginal and opportunity costs**

- Regarding comments related to the opportunity cost of water we used at least two definitions and in one exercise (competitive only thermal benchmark) we tried to isolate our study from any definition of opportunity costs.
- Opportunity costs do include implicitly CERE and FAZNI as reflects by agents bid and or marginal costs of thermal plants. There is no omission here.
- Regarding AGC and Ley 99. We don’t know if they will overturn the results (see introduction to this document).

- To calculate to marginal costs we used the fuel that was declared to XM.

- We have used a second set of marginal costs in the new report that use fuel prices reported by UPME. See section 4(c), page 47.

- Regarding the inframarginal rents we do not understand. Our conclusions are not based on absolute levels of bid price markups, but how they have change over time.

- Regarding the period of intervention and the use of liquid fuels see document. See section 6(c) page 95.

- We do not agree that the marginal cost of water has to be their bid. This ignores completely the possibly of abuse of market power. For an extensive discussion of this issue see Wolak study for New Zealand. An assessment of the performance of the New Zealand wholesale electricity markets. May 2009. Page 175.

**Information used doing calculations**

- Please see the new set of marginal costs based on UPME fuel prices (Methodology 2, marginal costs 2). See section 4(c) page 57.

- Changing AOM costs by 40% is unlikely to change our results. In any case it is an exercise worth doing.

- Using constant prices does not change our results (see section 6(d) and 6(e), page 97).

- We do not agree that the marginal cost of water is defined by their bid. This ignores the possibly of market power excercise.

**Markups**

- Ignoring the period in which the Government intervened the market does not change our results see section 6(d), page 97.

- Regarding the use of other variables as explanatory variables, for the spot price, we don't know (refer to the introduction to this document for a general comment on this claims). What we do provide is a validation exercise that has given us confidence in our results. We also specified and estimated several variants of the model and we settle down on this one as our best alternative.

- We discuss incentives to manipulate the technical parameters in our report. See section 8, page 120, for more details and a more general analysis. In any case what we have claimed is that there is a tradeoff associated with manipulating parameters but it is not obvious if there are incentives to do so. Therefore, we consider this is an empirical question that we analyze more thoroughly in this new version of the report.

- We do use "aportes hidricos" in our econometric analysis. This is used as an instrument and results are reported in page 87 and Appendix (e).

- The problem of endogeneity is a standard problem in econometrics and there are several techniques to deal with it.[2] The most common of them being the use of instrumental variables. Although we tried to use three variables as instruments, two of them turn out not to be good instruments therefore, we settle down on the variable that we ended up using and that most likely has reduced the endogeneity problem.

[2] Econometric Analysis of Cross Section and Panel Data. Second Edition. Woldridge, J (2012). Chapter 5.

**Welfare**

- The claim about aggregate production is something we actually checked and this is why we report actual and simulated production in section 5(e).

**Econometric model**

- Regarding the estimation of the model, we could avoid the period of intervention (see introduction).

- How good and useful is to have a high R^2 depends on the objective of the study. We are not studying volatility so we don't think that a high R^2 is fundamental for studying averages over years as we are doing. Also, one cannot assume that there is a model with a higher R^2 that resemble the output decision function of a rational agent but the existence of such an alternative model must be proven (see introduction).

**Structural model**

- It is hard to understand this section but there seems to be a misunderstanding regarding the role of the structural model. There are some claims that are difficult to support and that, to the extent of our knowledge, addresses problems that have not been solved either in theory or in practice. Actually, one of the reasons to do this study was to evaluate if a centralized dispatch improves efficiency. Nevertheless it is claimed that:

  *El modelo que simule el mercado, no debe ser un modelo centralizado, ya que no captura la percepción de riesgo de los agentes, considera que tiene información simétrica y perfecta y no las características propias del mercado.*

  We think that the introduction to the final report tries to explain the rationale for centralized unit commitment, its problems, and why we think it is an empirical problem and why there is no obvious answer for the questions raised. We don't agree with the claim that it assumes perfect and symmetric information, etc.

- The model optimizes hourly production costs plus startups for 24 hours. This is clear from the setup.

- We don't understand the comment about the international benchmarking. Our point was actually to emulate the XM model and not to use an international benchmark.

***Matriz energetica Colombiana***

- See introduction.

- Regarding demand response, see conclusion of the final report and Appendix G.

**Conclusions and recommendations of study**

- We have recommended a more in-depth study of competition.

- Regarding consumer's exposure see section 6(e).

- We completely disagree that we should not comment on market power. We think that our conclusions are valid and reinforce each one of the conclusions presented in this final version of the report.

**B. Specific Comments**

- Market power is an economic concept and used in positive economic analysis. It is NOT a value of judgment (see footnote 34, page 88).

- Regarding the gap between firm energy and aggregate demand, see section 4, pages 35, 36.

## Answers to ANDEG

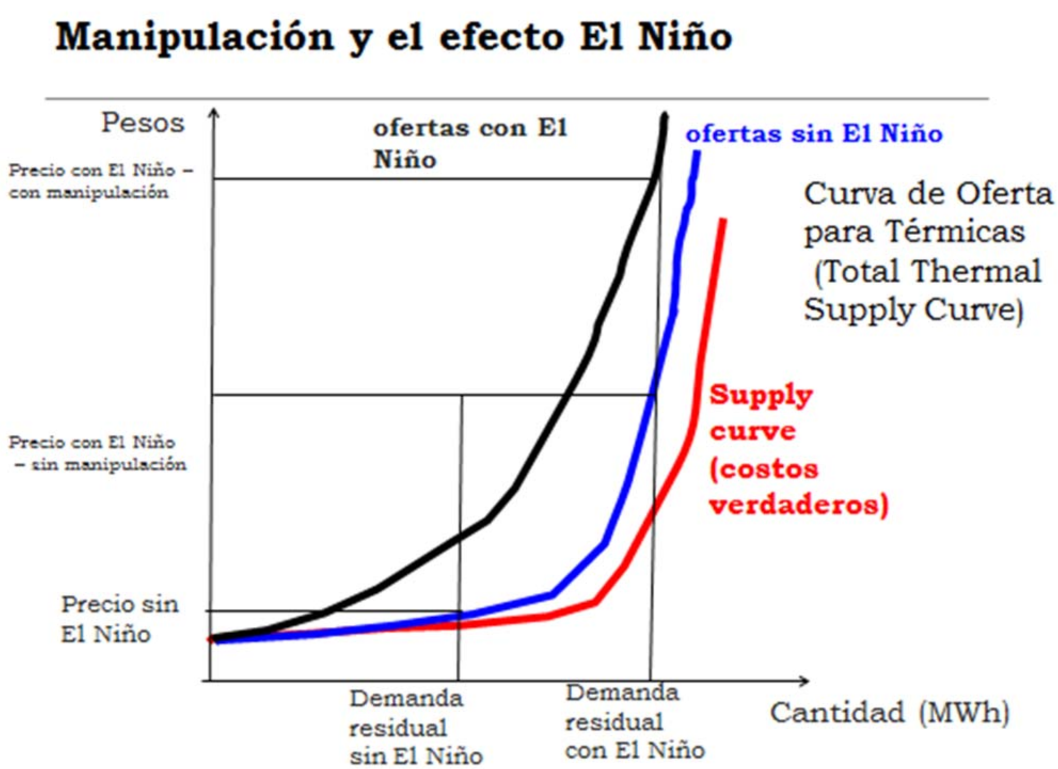
This document addresses on theoretical grounds, two main issues related to our study. First, it tries to establish some reasoning's about Nash equilibria and its implications for our conclusions. Overall we believe that the document misrepresents game theory, auction theory and how they apply to electricity markets. Second it reflects a complete misunderstanding of our second methodology for constructing a competitive benchmark.

- The appendix to this document (mostly based on de Castro and Riascos (2009). Characterization of bidding behavior in multi-unit auctions. Journal of Mathematical Economics) explains the two main paradigms of economic theory and lays down an auction model suitable to the context of electricity markets. We would like to stress that game theoretic framework that is used to analyze auctions is the theory of games with incomplete information and therefore, the central solution concept is that of Bayesian Nash equilibria, something apparently ignored in some of the comments of Andeg. This explains for example, why the discussion on whether the current mechanism is well implemented or not, is incorrectly framed theoretically. Without getting into the discussion illustrated in figure 1 (of their comments) if it is correct or not, it misses the point. If anything, agents in electricity markets are best described as playing a Bayesian Nash equilibrium which is not necessarily an expost Nash equilibria as the document suggests.

- The reference to Buchanan and Binmore is easily rationalized in our model of auctions (Appendix). In it, we can identify opportunity costs with the true valuation of water for example, and as we show and is well known in the specialized literature, optimal bidding is not truthful. Therefore, we are more than aware of these phenomena. A Nash (or Bayesian Nash) equilibrium in the game paradigm does not mean that it is competitive, unless the number of players is very large. On the contrary, a Nash Equilibrium reflects exercise of market power and there is an emerging literature on the "Cost of Anarchy" measuring the ratio of Nash Equilibrium Welfare to the Social Welfare solution. Furthermore the HHI index that attemts to quantify the level of market power as a function of market concentration is based on Nash Cournot equilibrium. Hence, leaving aside the positive theory of behavior in electricity markets, from a normative viewpoint the objective of the regulator should not be to induce a Nash Equilibrium but rather to achieve the outcome of a competitive Equilibrium that can be implemented through a Walrasian Auction. Submitting supply and demand functions can be viewed as a proxy to conducting a Walrasian Auction which is too costly but that is what is intended. In a Walrasian Auction players do not submit price bids but rather specify the quantity they are willing to produce when the Auctioner announces a price and likewise consumers specify how much they want to buy at each price. The Auctioneer moves the price until supply and demand clears. The Walrasian Auction is the paradigm we are trying to emulate but because of the complexity of doing it we are asking participants to submit proxies in the form of supply and demand functions or bids and offers that describe their quantity response for each price that the Auctioneer may announce. Under that paradigm, given that the market is competitive enough it is optimal for each participant to reveal true cost and willingness to pay.

- The arguments above explain why we don't focus on bid price markup absolute levels but rather on how the markups change over time. Something that the comments of most agents seem to be missing. Our point is not that agents should bid their marginal costs or opportunity costs but that (1) Variation in bid mark ups across time should have a good reason and (2) That to the extent that there is market power[3] there is space for improvement in terms of efficiency and (3) Given the evidence of market power, the regulator should do their best effort to stir agents towards a competitive outcome.

- Actually, during the presentation in November 25th, we showed the graph depicted in Fig. 1 below. That graph illustrates very clearly that we do not believe that bids should be equal to costs and that our conclusions are not based on such an a assumption. In this graph, the red curve represents the marginal costs of the thermal plants. The blue curve, which is above the cost curve, represents the bidding behavior that is expected in normal market circumstances. However, we found evidence---explained in our report--- of market power during ``El Nino'' events.[4] This would shift the actual bidding behavior to the black curve, way above the expected blue one. This suggests that El Nino could be acting as a coordination device for collusion among thermal plants, leading them to exercise their market power in a coordinated way.  It would be interesting to do the same econometric exercise we did using *aportes hidricos.*




[3] *…the ability to alter profitably prices away from competitive levels*. Mas Colell, Whinston and Green (1995)
[4] Simlar results have been obtained by Wolak (see reference above).

- The following paragraph:

  *Por lo anterior, consideramos equivocada la afirmación por parte de los consultores, al afirmar que los agentes, como robots, para demostrar su comportamiento competitivo deben ofertar el costo marginal de la planta y no valorar de ninguna manera el costo de oportunidad de los recursos de acuerdo con la información disponible en el mercado. Estamos seguros que el consultor considera que lo que opera en Colombia es una bolsa de costos, como la que opera en Brasil o Ecuador, por lo que para ese análisis aplicaría la conclusión manifiesta.*

  reveals a complete misunderstanding of our methodology. Nowhere in the document is it suggested that agents should bid marginal costs or opportunity costs (defined as mentioned in the previous paragraph). Our competitive benchmark is a *benchmark*, useful as a reference to compare against reality. What matters is how reality changes relative to the benchmark.[5]

- We did not use arguments that rely on the convexity of costs functions. Stoft's quote (reference not included) talks about caution in using the price signals in non-convex situations as a sufficient tool for long-term investments. However, we did not discuss long-term implications of prices in our report. Regarding the last paragraph. An in-depth thinking about our results show that it is logically possible that efficiency today (centralized unit commitment) has improved relative to what there was before (self-unit commitment) but that today we are further away from the competitive benchmark than we were before. Although paradoxical absolutely possible from a logical point of view. Time constraints have limited out ability to discuss this point further but the result does not cast doubts on our conclusions. Once again we see a complete misunderstanding of basic economic concepts. Yes indeed productive efficiency might have increased and at the same time markups may have increased. There is no flaw at all: assume that plants bid their marginal costs and opportunity costs. Then the dispatch is efficient and there is no markup. Now assume all plants bid their marginal costs or opportunity costs plus $100 KWh. Then the dispatch would again be efficient but now markups are significantly high. Furthermore, if demand is inelastic such an outcome would even be socially efficient since the markup results in no welfare loss but there would be a significant transfer of wealth from consumers to producers. With elastic demand, higher prices result in allocative inefficiency that reduce social welfare but productive efficiency gains may still be high enough to result in increased social welfare.

[5] The claim that the consultants consider that in Colombia operates a costs based dispacth is unfunded.

- We recognize that our study may have many weaknesses and that it is possible, although we believe very unlikely, that our main claims will be overturned by future studies. In any case, if that happens we are sure it will be completely unrelated to the comments raised in this specific letter.

## Comments by EPM, ISAGEN, EPSA, GECELCA

Comments by all these agents have been equally helpful to understand and improve our study. Nevertheless, we think that most, if not all of their comments have been answered implicitly in our new version of the report or more explicitly in this document. Actually many comments are exactly the same as those presented by Acolgen.

## David Harbord and Nils-Henrik von der Fehr

Comment:

Much of the report's analysis is sensitive to estimated hydro opportunity costs which are difficult to calculate, but the method chosen here is likely not very reliable outside of normal conditions. In any case, hydro opportunity costs should probably reflect more El Nino/ La Nina effects than they do in the report (during El Nino periods hydro costs might be near the value of lost load, or the Scarcity Price, and during Nina periods they are perhaps zero). A different specification of hydro costs to reflect Colombia's more extreme weather conditions could potentially alter some of the reports conclusions on welfare effects and overproduction by hydro versus thermal plant, for example. Further analysis of the sensitivity of the report's conclusions to alternative approaches to evaluating hydro costs therefore seems desirable.

Answer:

We used two alternative methods, one taking the hydro dispatch as it was just optimizing the Thermals and the other method pricing the hydro at opportunity cost. In the second method our opportunity cost was a low estimate except for periods of Nina (which in this data set was not a significant factor). Increasing the assumed cost of hydro to VOLL during El Nino periods or to the lowest rejected thermal bid (as proposed by CREG) whould only reinforce our conclusion that the hydro was over used and the thermal units underused.

Comment:

The report's welfare analysis ignores allocative inefficiency by assuming perfectly inelastic demand. This may be appropriate for a short-run (i.e. daily) auction analysis, but not for a longer-run welfare analysis. Medium and long-run electricity demand elasticity estimates for the USA and other countries are strictly positive, and often exceed 0.6. Including this in the welfare analysis might imply that overall efficiency and welfare were reduced by the 2009 Regulation 051, contrary to the report's conclusions.

Answer:

Even if we assumed demand elasticity as suggested, Our results still demonstrate that Regulation 51 increased the productive efficiency in the system by reducing dispatch cost but these gains have all been captured by the generators. It is possible that because of the exercise of market power by generators and increase in prices there was a loss in allocative efficiency. However, if the production efficiency gains would have resulted in lower prices as they should have allocative efficiency would have increased and so would have social welfare. Hence our conclusion that Regulation 51 has increased social efficiency in the competitive benchmark is valid if demand is elastic.

Comment:

The report concludes that price-cost margins have increased since 2009, but doesn't attempt any analysis of why or how the changes introduced in Regulation 051 might have led to the greater exercise of market power by generators. In the absence of such an analysis, or at least a deeper understanding of this issue, recommendations for adopting "a richer bid specification that more closely resembles actual cost structures", or particular market power mitigation measures, are not well founded and in our view premature.

One of our recommendations is that a deeper study be conducted to understand why price margins have increased and why generators are able to get away with higher markups. The suggestion that the mark ups reflect risk aversion by generators due to uncertainty in being able to recover their average energy production cost is speculative and gives generators the benefit of the doubt. A more detailed bid structure would eliminate such a motive and would facilitate truthful bidding if generators did have market power.

For example, with pure energy (pre Res 051 2009) bids, thermal generators bidding high prices into the auction risked being despatched discontinuously or for short periods of time, and thus not recovering their start-up costs. (In the 1990's, nuclear plant used to bid zero or near-zero prices into the UK pool to avoid this outcome, thus ensuring that they operated as baseload plant). Under the post- Res 051 2009 system, this risk was removed. So bidding in higher energy costs to set higher marginal prices would have become less risky and presumably more profitable. Further reducing risk by allowing for even more complex bids could conceivably make things worse rather than better.

Answer:

The notion that putting generators at risk can mitigate market power and promote competitive behavior is ill conceived and has no theoretical or empirical foundation. Our recommendation to increase the richness of a bid format was never intended to mitigate market power but rather to increase transparency so that market power abuse can be more easily monitored. In a competitive setting generators are motivated to sell energy at any price above marginal cost as long as they are assured that their startup and total energy cost is covered. In a uniform price setting if a generator thinks that by raising the price it can affect the market clearing price then it does not behave as a price taker and it must have market power to be successful in pursuing such a strategy. In such a case the market is not competitive and market power mitigation and active intervention is appropriate in order to ensure efficiency.

On the other hand we also learn from the theory of supply function equilibrium that a supply function equilibrium with general non linear supply functions is more competitive than a supply function equilibrium that is constrained to linear supply functions. In other words in an oligopoly market a bid structure with more degrees of freedom allows suppliers to compete more aggressively against each other to the benefit of consumers and it is more difficult for them to collude or coordinate their actions. (See paper by Hogan and Baldick on supply function equilibrium and paper by Oren, Wilson and Smith, "Competitive Nonlinear Pricing").

Comment:

We can't say if the points immediately above are all correct, but merely wish to point out that in the absence of an analysis of these issues, adopting one or more new measures to solve the problems identified in the report is likely to be counterproductive. To give thus point further weight, recall that Frank Wolak's 2009/10 study found that the introduction of two-part bidding in Res 051 2009 did not reduce energy (variable cost) bids nor improve dispatch efficiency. 1 He also thought that hydro resources were being underutilized during peak periods, and recommended introducing "multiple offer price steps for each generation unit that are fixed for the day" to solve this, in contrast to the current report's findings and recommendations. Hence these two studies are not in complete agreement on either the problems or the solutions.

However, Wolak's cautioning against the introduction of more complex bids to guarantee start-up cost recovery now sound prescient. 2 As he observed, "suppliers that receive guaranteed start-up cost recovery still have strong financial incentives to exercise all available unilateral market power in setting their energy offer prices." And as we have noted above, guaranteeing start-up recovery may have actually increased incentives to exercise market power (i.e. increase price bids) by eliminating the risks entailed by pure energy bids.

Answer:

Wolak's analysis was based on a data set from 2009/2010 which was heavily influenced by the El Nino phenomenon. Like him we concluded that Resolution 51 did not reduce marginal energy bids. However, we looked at a larger data set and do observe efficiency gains.

We did not claim that Resolution 51 or a more complex bid structure will control Market power. Such modifications, however, will better expose market power abuse since they will remove the excuse of risk as a reason for marking up prices. The only reliable way to control market power is market intervention when such abuse takes place and a bid structure that enables generators to reflect their true cost structure and minimize their risk makes it easier to monitor market power abuse and mitigate it.

# Appendix:

# A Note on Games and Auctions in Electricity Markets

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December 20, 2013

## 1 Two basic frameworks in Economic Theory: Competitive and Strategic

Economic Theory has evolved since the last century around two main paradigms: competitive markets and Game Theory.[1] The competitive market paradigm achieved a high state of development with the major contributions of Arrow and Debreu in 50's and 60's. Game Theory main contributors were von Neumann in 1940's and John Nash, in the 50's. Most of the ideas used to analyze electricity markets nowadays can be traced back to these two paradigms. Since they refer to different situations, it is useful to distinguish these two paradigms, for being able to identify claims that are related to one or another.

The competitive markets framework assumes that economic agents do not have any possibility of affecting prices, that is, they are "price-takers". This is a good approximation in big markets, with small consumers, although some markets do not follow into this category. The famous supply and demand curves belong to this paradigm. The *short-run* supply curve is given by the marginal costs of the producers, and do not include fixed or sunk costs. It is possible to consider *long-run*

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[1] More recently, in 1980's, another revolution took place, around information and its strategic use. We will comment about this later.

equilibria, in which case the supply curve should include fixed costs and, if the market is expanding, also sunk costs. Notice that other implicit assumptions in this framework are that there is no private information and the product is homogenous among producers.

Depending on the specific electricity market, these assumptions may *approximately* hold. Usually each firm has some possibility of setting the price, which means that the assumption of "price-takers" does not strictly hold. However, if there are many different firms, the probability of setting the price is small, and this framework might be suitable for analysis. The fact that costs are known is also approximately true, but not completely. For instance, generators may have negotiated different private contracts, that allow them to use fuel at a different price than the current spot price for that fuel. This changes their marginal costs and, therefore, the supply curve. In general, this is not a problem if firms are price-takers, but if they have some possibility of setting the price and may submit bids that are different from their true costs (as it happens in real markets), they might misrepresent *strategically* their costs, which may lead to an inadequacy of the competitive markets framework and the need for considering the strategic framework embodied by Game Theory.

In the more realistic setup of Game Theory, economic agents have a proper role in the definition of the outcome of the economic interaction. While in the competitive benchmark, they are passive "price-takers," in a game they play actions and the profile of all players actions (together, sometimes, with some random shock) define the outcome of the game. Each agent has an utility that depend on this outcome and tries to choose the best action to maximize this utility. Section 2 includes further discussion about this framework, including the solution concept commonly adopted in this framework: the Nash equilibrium.[2]

It is clear that Game Theory is closer to reality, but this realism comes at a price. The more realistic a model, the harder to fully work out its conclusions. Thus, while the competitive markets frameworks lacks in realism, it provides a really simple benchmark for analysis. Game theoretic models are usually more realistic but may require analyzes that might prove unpractical.

Another layer of complexity, but certainly in the direction of more realism is to consider games where the players have some private information. This is relevant in electricity markets, as not only firms have private knowledge about their costs (through private contracts, for instance), but the

[2]Or more precisely, when there is private information: Bayesian Nash equilibrium.

agents also have independent assessment of future conditions (as availability of water in the future, for instance) and this information might be extremely useful to the market as a whole and it is actively used by market participants. Thus, aggregating this information in the game formulation is important. When we do this, in general we enter the realm of Bayesian Games, of which auctions are a special and important sub-class.

Fortunately, there is a link between Bayesian games, like auctions, and competitive markets. Indeed, it has been established by countless papers that auctions tend to converge to competitive markets when the number of participants grows to infinity in a stronger rate than the number of units on sale.These results on convergence establish a link between the two paradigms and offer the comfort that their conclusions are not necessarily in conflict.

Thus, there are different paradigms in state-of-art Economic Theory, that might be used in the analysis of electricity markets. In general, economists use both of them, but it is not always clear, to the unprofessional reader, what framework is mentioned. Of course these are frameworks for analysis, and therefore have their limitations. The reasoning based on these framework usually guide policy decisions and it is therefore useful to recognize them, to avoid confusion in the arguments and conclusions. In the next section we examine with a little bit more of detail the more complex of these frameworks, the game theory one with private information, and discuss some of its features.

## 2 A Short Overview of Game and Auction Theory, with a Discussion about its Applicability to Colombian Electricity Markets

Game theory allows the study of strategic iterations, when the action of one agent affects the outcomes for a set of players. Such situations are called games and are formalized by specifying the following elements:[3]

- the set of players;
- the action set available to each player;

[3]For simplicity, we will restrict attention to simultaneous games, with no history or information sets about past actions.

- the utility, depending on the actions played and some other random shock;
- the information available to each player (in case it is not a game of complete information).

A formal analysis of a game requires a proper clarification of these elements. In the case of electricity markets, and specifically the day-ahead market that exists in Colombia, some of these elements seem easy to define, as the set of players (generators).[4] However, there are other elements that are not so obvious and may require a modeling choice. For instance, we may restrict the game just to the bids subjected each day; or we may consider more complex actions, that cover many days in such a way as to allow taking advantage of the ramp restrictions that connect production in one day to production in other days. We can even go further and include in the specification of the action sets, the information about ramp and costs constraints provided every three months and also the possibility of scheduling tests. As we can see, a realistic description of the whole game is much less simple than may appear to a non-specialist, at the beginning.

Another complication arises with respect to the modeling of information. In the case of electricity markets, the private information is related to costs. It is true that costs are known *to some certain extent*, but cannot be perfectly known. Not only the costs of fuel are given by contracts that are not publicly known, but the efficiency of each turbine are known only approximately. Moreover, when there is some kind of failure, or the need for some maintenance, the costs can change dramatically.

The usual way to model private information is through probability models. We assume that the privately known variable is a random variable, obeying a given distribution. We illustrate this procedure below.

Once all model elements are specified, we can then define Nash equilibria—or Bayesian Nash in the case we have private information—as a profile of strategies such that the strategy specified for each player is optimal when all other players are playing the strategies specified in the profile.

In the remaining of this section we introduce a standard formalization of an auction game, as developed by de Castro et;a. (2009). The purpose of this is just to illustrate some of the theoretical results that are available in the Auction Theory literature. This model would require some adaptations for the application to the Colombian electricity markets, but *these adaptations*

[4] Even this can be questioned: in some circumstances, it may be better to ignore the hydro generators and consider the game between only thermal generators.

*are outside the scope of this project.* The purpose is just o illustrate some of the classical results in auction theory that would be useful to clarify the issues that Andeg's letter raised.

## 2.1 Players and Information

In this subsection, we describe an auction game among $N$ strategic bidders. We denote by $\mathcal{N} = \{1, ...N\}$ the set of all players. Player $i \in \mathcal{N}$ receives a signal (i.e., private information), $t_i \in T_i$ where $T_i$ denotes player $i$'s information set. We denote by $t = (t_1, t_2, ..., t_N) = (t_i, t_{-i})$ the vector of all players' information, where $t_{-i} = (t_1, ..., t_{i-1}, t_{i+1}, ..., t_N)$, as usual. Let $\Im_i$ be a $\sigma$-field of subsets of $T_i$ and define $T \equiv_{i \in \mathcal{N}} T_i$ and the product $\sigma$-field over $T$, $\Im =_{i \in \mathcal{N}} \Im_i$. Players beliefs are functions $\tau_i : T_i \to \Delta(T_{-i}, \Im_{-i})$, where $T_{-i} \equiv_{j \neq i} T_j$, $\Im_{-i} =_{i \neq j} \Im_i$ and $\Delta(T_{-i}, \Im_{-i})$ is the set of probability distribution over the measurable space $(T_{-i}, \Im_{-i})$. If $g$ is a function of $t_{-i}$, we denote the expectation of $g$ with respect to $\tau_i(t_i)$, by $E[g|t_i]$. Also to ease notation we will write $\tau(\cdot|t_i)$ instead of $\tau_i(t_i)$.

Notice that we do not assume that players beliefs need to represent conditional beliefs derived from a common prior over $T$. Individual signals may be dependent and of arbitrary dimension.

## 2.2 Action sets and outcomes

The bidders have to attend a demand of $K$ identical indivisible objects. Each player $i \in \mathcal{N}$ comes to the auction with $e_i \in \{0, 1, 2, ...\}$ units of the same object, and $\sum_{i=0}^{N} e_i = K$. After receiving its signal, a strategic player submits a sealed proposal, that is, a bid (or offer) that is a vector of real numbers, $b_i \in B \subseteq \mathbb{R}^K$ where $B$ denotes the set of valid bids, that is, $B = \{b \in \mathbb{R}^K : b_k \geq b_{k+1}$ for $k = 1, ...K-1\} \cap [\underline{b}, \overline{b}]$, $b_{i,k}$ is the maximum value that bidder $i$ is willing to pay for the $k$'th unit, given that he is receiving $k-1$ units; and $[\underline{b}, \overline{b}]$ denotes a $K$ dimensional rectangle that bounds the set of all bids. Since bids are non-increasing we are implicitly assuming that there are no complementarity among objects. Bids are in units of account (i.e., dollars). The non-strategic player 0 also places a bid $b_0 \in B$, meaning that there is a reserve price for each unit.[5] For instance, in a one-object auction ($K = 1$) where all players are buyers, if $\max_{j=1,...,N} b_{j,1} < b_{0,1}$, this means that none of the bidders are willing to pay the reserve price. The difference is that $b_0$ is known to

[5] If the model does not specify a reserve price it is usual to assume $b_0 = 0$.

everyone at the time the auction takes place, while $b_j$, $j \neq 0$, is not known to bidder $i \neq j$, $i \in \mathcal{N}$.[6] We denote by $b$ the vector of all players' bids, $b \in \mathbb{R}^{(N+1)K}$.

## 2.3 Allocation and Payoffs

The "auction house" computes the bids and determines how many units each player receives. If player $i$ wins a $k$'th unit, his payoff is increased by $u_{i,k}(t, b)$, where $u_{i,k} : T \times \mathbb{R}^{(N+1)K} \to \mathbb{R}$.[7] Thus, if player $i \in \mathcal{N}$ ends the auction with exactly $m_i \in \{0, 1, ..., K\}$ units, his payoff is $\sum_{k=0}^{m_i} u_{i,k}(t, b)$. The utility functions $u_{i,k}(t, b)$ incorporate in their definition the number of units $e_i$ that bidder $i$ has, as we explain in the examples below. If $k \leq e_i$, $u_{i,k}(t, b)$ stands for the utility of keeping the $k$-th object and if $k > e_i$, $u_{i,k}(t, b)$ stands for the utility of buying the $k$-th object. We will explain below the allocation rule, which will complete the definition of the payoff for each bid profile. In the examples we shall restrict to separable transfers so, for later reference, for each player $i$ and unit $k$, let $v_{i,k} : T \to \mathbb{R}$ be a function such that $v_{i,k}(t)$ represents the (marginal) value, in units of account, of the $k$'th unit for player $i$ when the vector of signals is $t \in T$.

If $m_i < e_i$, the player has sold $e_i - m_i$ units in the auction and if $m_i > e_i$, the player has bought $m_i - e_i$ units in the auction. No negotiation was made if $m_i = e_i$.

Given $b_{-i}$, let $s_i = (s_{i,1}, s_{i,2}, ..., s_{i,K})$, with $s_{i,1} \leq s_{i,2} \leq ... \leq s_{i,K}$, denote the (inverse) residual supply curve facing bidder $i$. That is, $s_{i,K}$ is the highest of the bids by players $j \neq i$, $s_{i,K-1}$ is the second highest and so on. Thus, for getting (for sure) at least one unit, bidder $i$'s highest bid must be above $s_{i,1}$, that is, $b_{i,1} > s_{i,1}$. For bidder $i$ winning at least two units, it is necessary $b_{i,2} > s_{i,2}$ and so on. Figure 1 illustrates this.

[6] Unknown reserve prices can be modeled as the bid of a strategic bidder.

[7] We consider the dependence on $b$ instead of $b_i$ because we want to include in our results auctions where the payoff depends on bids of the opponents, such as the second-price auction, for instance. Also, this allows the study of "exotic" auctions, i.e., auctions where the payment is an arbitrary function of all bids.

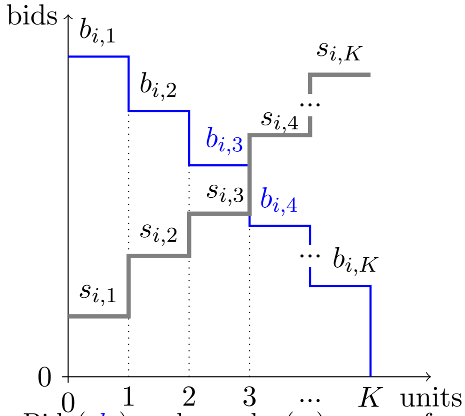


Figure 1: Bid ( $b_i$) and supply ($s_i$) curves for bidder $i$. In the situation displayed, bidder $i$ receives three units, because $b_{i,3} > s_{i,3}$ but $b_{i,4} < s_{i,4}$.

In order to decide who wins an object, we will assume that the auction house uses an allocation (or tie-breaking) rule.

**Definition 1** *An allocation rule is any function* $a : \mathbb{R}^{(N+1)K} \to [0,1]^{(N+1)K}$ *such that:*

1. *If* $b_{i,k} < s_{i,k}$ *then* $a_{i,k}(b) = 0$.
2. *If* $b_{i,k} > s_{i,k}$ *then* $a_{i,k}(b) = 1$.
3. *The allocation rule is non-increasing in* $k$*, that is, for all* $k' \leq k$, $a_{i,k'}(b) \geq a_{i,k}(b)$.[8]
4. ${}_{i=0}^{N}{}_{k=1}^{K} a_{i,k}(b) = K$.

The interpretation is the following. If $a_{i,k}(b) = 1$ then player $i$ wins at least $k$ objects. If $a_{i,k}(b) = 0$ then player $i$ wins at most $k-1$ objects. Formally, the first condition says that if player $i$'s $k$-th bid is lower than the $K-k+1$ highest competing bid he will not be awarded the $k$-th object. The second condition says that if player $i$ bids higher for unit $k$ than the $K-k+1$ highest competing bids then he will win at least $k$ objects. The third says that if he wins at least $k$ objects then he must also win at least $1, ...k-1$ objects. The fourth says that at most $K$ units are allocated among the $N$ agents.

[8]If there is no tie, this condition follows from 1 and 2.

Observe that in the definition of allocation rules, there is freedom to define the rule only when $b_{i,k} = s_{i,k}$, provided the other conditions are satisfied. Thus, it is sufficient to define the rule for ties.

This setting is very general and applies to a broad class of discontinuous games. For a list of examples, see de Castro et.al (2009).

The Colombian electricity market would require further specifications and some adaptations, such as the requirement that the only one price be offered for all quantities and the introduction of constraints for the selection of feasible bids. Nevertheless, this setup allows us to present some results that can guide the analysis and the answers to Andeg, developed in the next section.

### 2.4 Theoretic Results

After developing what they call the Basic Principle of Bidding, de Castro et.al (2009) obtained the following results that are useful for later discussion.

**Example 2** *(Multiple Object Discriminatory Auction). Let $u_{i,0} = 0$, $u_{i,k}(t,b) = v_{i,k}(t) - b_{i,k}$. Then $\partial_{b_{i,j}} u_{i,k}(t,b) = 0$ if $j \neq k$ and $-1$ if $j = k$. Then the optimal bid obeys the following condition:*

$$b_{i,k} = E[v_{i,k}(t) | t_i, \mathbf{s}_{i,k} = b_{i,k}] - \frac{F_{\mathbf{s}_{i,k}}(b_{i,k}|t_i)}{f_{\mathbf{s}_{i,k}}(b_{i,k}|t_i)}.$$

*Notice that misrepresentation of values is an immediate consequence of this characterization. Indeed, the optimal bid is below the value (the first term on the right), since the second term is negative.*

In an electricity auction, this means that the optimal bid is above the costs, that is, in a discriminatory auction, the optimal action is to misrepresent the true costs. This result is not valid if we use Vickrey Auctions, as shown below.

**Example 3** *(Multiple Object Vickrey Auction). Let $u_{i,0} = 0$, $u_{i,k}(t,b) = v_{i,k}(t) - \mathbf{s}_{i,k}$. Then $\partial_{b_{i,j}} u_{i,k}(t,b) = 0$. **?** show that the optimal bid obeys the following condition:reduces to:*

$$b_{i,k} = E[v_{i,k}(t) | t_i, \mathbf{s}_{i,k} = b_{i,k}].$$

*Therefore in a general Vickrey Auction, an optimal bid is truthful.*

The Colombian electricity market uses a uniform price auction. Therefore, the following result is more relevant.

**Example 4** *(Uniform price auction) Let $u_{i,0} = 0$, $u_{i,k}(t, b) = v_{i,k}(t) - p$, where $p$ is the payment, which is equal for all units and bidders. There are two common rules for the uniform price auction. One is the highest looser bid, which is the uniform price auction described by Krishna (2002). In this case, the payment is equal to the highest bid among those bids that do not receive the object. A variant is to put the payment equal to the lowest winning bid. We treat both below. Note that for any $k$,*

$$\partial_{b_{i,j}} u_{i,k}(b_{i,j}, \cdot) = -\partial_{b_{i,j}} p(b) = \begin{cases} -1, & \text{if } b_{i,j} \text{ determines the payment} \\ 0, & \text{otherwise} \end{cases}$$

*In the case of the lowest winning bid, $b_{i,j}$ determines the payment in the event $s_{i,j} < b_{i,j}$ and $b_{i,j+1} < s_{i,j+1}$.*[9] *This event is contained in the event $[b_{i,k} > \mathbf{s}_{i,k}]$ if and only if $k \leq j$. Thus, the first order condition becomes:*

$$b_{i,j} = E\left[v_{i,j}(t) | t_i, \mathbf{s}_{i,j} = b_{i,j}\right] - j \frac{\Pr\left[s_{i,j+1} > b_{i,j+1}, b_{i,j} > s_{i,j}\right]}{f_{\mathbf{s}_{i,j}}(b_{i,j}|t_i)}.$$

*In the case of the highest loosing bid, $b_{i,j}$ determines the payment if $b_{i,j} < s_{i,j}$, $b_{i,j-1} > s_{i,j-1}$ and $b_{i,j} > s_{i,j-1}$. Similarly,*

$$b_{i,j} = E\left[v_{i,j}(t) | t_i, \mathbf{s}_{i,j} = b_{i,j}\right] - (j-1) \frac{\Pr\left[s_{i,j+1} > b_{i,j+1}, b_{i,j} > s_{i,j}\right]}{f_{\mathbf{s}_{i,j}}(b_{i,j}|t_i)}.$$

Notice that misrepresentation of values is an immediate consequence of the above characterization. This can be generalized in the following.

### 2.4.1 Sufficient Conditions for Truthful bidding

In the de Castro et.al (2009) we show the following.

**Proposition 5** *Assume the conditions of Corollary 1 in de Castro et.al (2009) and (1) The bid $b_{i,j}$ never modifies the payment of any unit, then it is optimal for bidder $i$ to bid $b_{i,j}$ such that:*

$$E\left[u_{i,j}(\cdot) | t_i, \mathbf{s}_{i,j} = b_{i,j}\right] = 0.$$

[9] We do not consider situations where two bids are equal.

*(2) In addition to the previous condition, assume that* $u_{i,j}(t,b) = v_{i,j}(t) - p_{i,j}(b)$*, and that the payment* $p(b)$ *is* $b_{i,j}$ *in case of a relevant tie at* $b_{i,j} = s_{i,k}$*. Then the optimal bid is to bid the expected value of the unit:*

$$b_{i,j} = E\left[v_{i,j}(t)\,|t_i, \mathbf{s}_{i,j} = b_{i,j}\right].$$

***Proof.*** *(1) It is sufficient to examine the expression of* $\partial_{b_{i,j}}\Pi_i(t_i, (\beta, b_{i,-j}), \mathbf{b}_{-i})$*. (2) Observe that in the conditional event* $\mathbf{s}_{i,j} = b_{i,j}$*, the payment is* $p(b) = b_{i,j}$*.* ■

As a corollary, they obtain:

**Corollary 6** *The first (highest) bid in the uniform price auction (with payment equal to the highest looser bid) is truthful.*

The above result may suggest a rationale for using just one bid in Colombian day-ahead electricity market, but this would be a stretch of the result. A more detailed model of such market would be necessary to confirm the validity of such rationale. On the other hand, one-price bids can create allocation inefficiencies. Thus, even if there is indeed a "cost-revelation" rationale for the use of such format, it has implications on the efficiency side.